# Physics II <br> ISI B.Math <br> Final Exam : December 16,2020 

Total Marks: 80
Time: 3 hours
Alternate email of instructor for questions: sukanyasin@yahoo.com
Answer questions 1-4 and either 5 or 6

1. (Marks : $\mathbf{1 2}+4$ )

(a) An ideal gas is contained in a large jar of volume $V_{0}$. Fitted to the jar is a glass tube of cross sectional area $A$ in which a metal ball of mass $M$ fits snugly. The equilibrium pressure in the jar is slightly higher that the atmospheric pressure $p_{0}$ because of the weight of the ball. If the ball is displaced slightly from equilibrium, it will execute simple harmonic motion(neglecting friction). If the states of the gas represent an adiabatic quasistatic process, and $\gamma$ is the ratio of the specific heat at constant pressure to the specific heat at constant volume, find a relation between the oscillation frequency $f$ and the variables of the problem.
b) An insulating chamber is divided into two halves of volume. The left half contains a diatomic ideal gas at pressure $P_{0}$ and temperature $T_{0}$ and the right half is evacuated. A small hole is opened between the two halves and the gas is allowed to flow through and the system eventually comes to equilibrium. No heat is exchanged through the walls. Find the final temperature of the system.
2.(Marks: $8+2+6$ )

a) An ideal monatomic gas is taken around the above rectangular cycle shown in the above $P-V$ diagram. Let this operate as a heat engine to convert the heat added to mechanical work.
(i) Evaluate the efficiency of the engine. (ii) Calculate the efficiency of an "ideal" engine operating between the same temperature extremes.
b) The efficiency of a heat engine is to be improved by lowering the temperature of its low temperature reservoir to a temperature $\tau_{r}$ below the environmental temperature $\tau_{l}$ by means of a refrigerator. The refrigerator consumes part of the work produced by the heat engine. Assume that both the
heat engine and refrigerator operate reversibly.Calculate the ratio of the net (available) work to the heat $Q_{h}$ supplied to the engine at temperature $\tau_{h}$. Is it possible to get a higher net energy conversion efficiency this way?
2. (Marks : $5+4+3+4)$
(a) Derive the equation

$$
\left(\frac{\partial C_{V}}{\partial V}\right)_{T}=T\left(\frac{\partial^{2} P}{\partial T^{2}}\right)_{V}
$$

and show that $C_{V}$ for an ideal gas is a function of $T$ only
(b) Adiabatic free expansion of a gas is a process where the internal energy $U$ remains constant. In this context, the following quantities are of interest.
(i) What is $\left(\frac{\partial T}{\partial V}\right)_{U}$ ? Express the result in terms of $P, T,\left(\frac{\partial P}{\partial T}\right)_{V}$ and $C_{V}$.
(ii) What is $\left(\frac{\partial S}{\partial V}\right)_{U}$ ? Express the result in terms of $P$ and $T$.
(iii)From the above results find the change in entropy of one mole of an ideal gas when it expands freely and adiabatically from volume $V_{1}$ to $V_{2}$.
4. (Marks : $\mathbf{1 0}+\mathbf{6}$ )
(a) Consider an ideal gas with $N$ particles. Suppose the energy of each particle $\epsilon$ is proportional to the magnitude of its momentum, i.e, $\epsilon=c p$ (This is actually the relativistic form of energy of a particle which has a very high energy). Find the Helmholtz free energy, pressure and the heat capacity at constant volume for this gas. Ignore the internal structure of the particles.
(b) Consider a binary alloy system of two distinct chemical components $A$ and $B$ whose atoms occupy distinctly numbered sites. The entropy associated with assembling such an alloy is known as the entropy of mixing.
Show that the entropy of mixing for an alloy consisting of N atoms where $t$ atoms are of type $B$ and $x=t / N$ is approximately given by

$$
S(x) \simeq-N k[(1-x) \log (1-x)+x \log x]
$$

where $N$ is large.
5. (Marks : 12 +4 )

(a) In the Newton's ring arrangement if the incident light consists of two wavelengths 4000A and 4002A calculate the distance from the point of contact at which rings will disappear.This happens when the dark ring of one wavelength merges with the bright ring of another. Assume that the radius of curvature of the curved surface is 400 cm . In the same arrangement, if the lens is slowly
moved upwards, calculate the height at which the fringe system (around the centre) will disappear.
(b) What is the state of polarization when the $x$ and $y$ components of the electric field are given by
(i) $E_{x}=E_{0} \cos (\omega t+k z), E_{y}=\frac{1}{\sqrt{2}} E_{0} \cos (\omega t+k z+\pi)$
(ii) $E_{x}=E_{0} \sin \left(k z-\omega t+\frac{\pi}{3}\right), E_{y}=E_{0} \sin \left(k z-\omega t-\frac{\pi}{6}\right)$
6. (Marks : 6 + 4 + 6 )
(a) Two transparent slabs having equal thickness but different refractive indices $\mu_{1}$ and $\mu_{2}$ are pasted side by side to form a composite slab. This slab is placed just after the double slit in a Young's experiment such that light from one slit goes through one material and light from the other slit goes through the other material. What should be the minimum thickness of the slab so that there is a minimum at the point $P_{0}$ which is equidistant from the slits?
(b) Show that the first and second order spectra will never overlap when a grating is used to study a light beam of visible light.
(c) A grating (with 15000 lines per inch) is illuminated by sodium light. The grating spectrum is observed on the focal plane of a convex lens of focal length 10 cm . Calculate the separation between the $D_{1}$ and $D_{2}$ lines of sodium with wavelengths 5890 A and 5896 A respectively.

